Forecasting the Exchange Rate using Non-linear Taylor Rule Based Models

By

Rudan Wang (Business School, Coventry University)

Bruce Morley* (Department of Economics, University of Bath)

Michalis P. Stamatogiannis (University of Liverpool Management School)

Address for correspondence: Department of Economics, University of Bath, Bath, BA2 7AY, UK. Tel. +44 1225 386497, E-mail. bm232@bath.ac.uk, Fax. +44 1225 383423. This research did not receive any specific grant from funding agencies in the public, commercial, or not-for-profit sectors.

Forecasting the Exchange Rate using Non-linear Taylor Rule Based Models

1. Introduction

The aim of this study is to investigate whether the implementation of the Smooth Transition Regression (STR) approach to a Taylor rule model can offer substantial gains in the modelling and forecasting of the exchange rate. Since the start of the floating exchange rate era, a number of studies have attempted to explain exchange rate movements, although as suggested by Cheung *et al.* (2005), mostly with limited success. Explaining and predicting exchange rate movements is an important aspect of monetary policy, particularly as capital flows between international asset markets have increased. Recently, a new strand of the exchange rate literature has developed a series of models that combine interest rate reaction functions based on the Taylor rule and the exchange rate, which has produced more accurate forecasts (Molodtsova and Papell, 2009; Wang *et al*., 2016). These models tend to reflect more realistically how monetary policy has been conducted or evaluated over the recent past, and offers an alternative interpretation of exchange rate dynamics. Although these studies have provided evidence of short-term predictability (Molodtsova and Papell, 2009) the results tend to vary across countries and different time intervals.

Further recent developments in the study of exchange rates and monetary policy in general have involved the use of non-linear estimation techniques, which have become prevalent in both the literature on Taylor rule models (Qin and Enders, 2008) and modelling of the exchange rate (Michael *et al.*, 1997, De Grauwe and Grimaldi, 2005). In addition non-linear exchange rate models have also been used successfully for forecasting the exchange rate (Boero and Marrocu, 2002). As far as we know, there has as yet been no attempt to connect both strands of the literature and use the STR approach on the Taylor rule type exchange rate models, particularly with respect to forecasting.

The main contribution of this study is the estimation and forecasting of the Taylor rule based exchange rate model using the STR approach, in particular the Logistic and Exponential STR models, having pre-tested for non-linearity in the variables. We also incorporate wealth effects into the models to reflect the importance of asset markets to both the Taylor rule and exchange rate determination. This study uses real time output data for the estimation of the Taylor rule model and includes bilateral exchange rates for Australia, Sweden and the UK with respect to the US dollar. A number of different types of non-linear techniques have been used to model the exchange rate[1], in this study we use the STR models, which were originally applied to nonlinearities over the business cycle by Teräsvirta and Anderson (1992). Over the recent years, it has been applied successfully in many exchange rate studies including Purchasing Power Parity (PPP) by Michael *et al*. (1997) as well as monetary models and Uncovered Interest Parity (UIP) by Baillie and Kilic (2006). One appealing feature of the STR methodology is that it requires no prior information about the threshold level at which the model changes. Moreover, compared to other alternative nonlinear models such as the Markov switching model and threshold autoregressive model, the STR family of models allows for a smooth and gradual transition from one regime to another instead of sudden jumps between regimes.

[1] Examples of incorporating nonlinearities in the modelling of the exchange rate include Bollerslev (1990) who allows for time varying conditional variance and covariance of the error term, Engel (1994) who uses a Markov switching forecasting model and Rapach and Strauss (2008) who allow for structural breaks in the conditional volatility.

The theoretical basis for this study is the linear model of the exchange rate by Molodtsova and Papell (2009) which has been augmented by the inclusion of wealth effects, including both stock prices and house prices. As Case et al. (2005) suggest both have varying degrees of influence on the macro-economy. Other studies including Granger et. al. (2000) have analysed wealth effects in the form of stock prices and the exchange rate, finding a significant relationship.[2] Unlike most previous studies with the STR models, we have documented and compared results across different models with regard to a large number of macroeconomic transition variables including the real exchange rate, output gap, inflation difference, interest rate difference, wealth effect and a measure of exchange rate volatility. All transition variables are contemporaneous, apart from the interest rate differential, because exchange rates are determined in financial markets where we would not expect much of a lagged effect to occur. Some studies concentrate on a small number of transition variables and then use statistical inference to choose the specific lag for each transition variable. For instance, Lutkepohl *et al.* (1999) chose the variable with the smallest *p*-value in the context of a Lagrange Multiplier (LM) type test of linearity against STR. They test a number of potentially lagged and non-lagged transition variables, including different lags, finding a non-lagged variable to be the most appropriate. The choice of the most appropriate transition variable in this study is based on model specification and diagnostic testing. To motivate the use of the STR model, we first run a linearity test on the linear Taylor rule model.

As with similar studies such as Molodtsova and Papell (2009), the main emphasis of this paper is on the out-of-sample forecasting performance of the nonlinear STR type exchange rate models. However as with the exchange rate literature as a whole, the use of non-linear models for

[2] Wang *et* al. (2016) have demonstrated the robustness of the linear version of a wealth augmented Taylor rule based exchange rate model in terms of both the in-sample and out-of-sample performance relative to the original model.

forecasting has produced mixed results (Rapach and Wohar, 2006), with the performance relative to the benchmarks varying across different samples as in Pavlidis *et al*. (2012) or the degree of non-linearity in the data as in Enders and Pascalau (2015). In this study, the evaluation of the models' forecasting performance is conducted against different benchmarks including the random walk model and the linear Taylor rule based model. The results from estimation and forecasting of the nonlinear STR model provide evidence of the nonlinear relationship between the exchange rate and economic variables. Moreover, in our study, we have found evidence that the STR models outperform the random walk, a simple uncovered interest rate (UIP) model and the linear Taylor rule model in out-of-sample forecasting of the exchange rate.

Following the introduction, Section 2 introduces the STR model and the nonlinear Taylor rule exchange rate model and outlines the specification, estimation and forecast evaluation techniques used in this study. In Section 3, we estimate the models and compare their in-sample specification and out-of-sample forecasting performance. The main conclusions are then drawn in the final section.

2. Material and Methods

2.1 The Modelling Framework

We have used the Taylor rule as the basis for the exchange rate model as this type of model tends to explain more realistically how monetary policy has been conducted in practise over the recent past by most central banks. We have also used it because the linear version of this model has largely been more successful at forecasting the exchange rate (Molodtsova and Papell, 2009) relative to other exchange rate models, such as the monetary model. The theoretical Taylor rule (Taylor, 1993) assumes that the nominal interest rate depends on changes in inflation and the output gap. Our starting point is the forward-looking Taylor rule (Clarida *et al*., 1998), where we assume the foreign country targets the exchange rate in its Taylor rule and the interest rate is assumed to adjust gradually towards its target level. In addition to this original specification, this study extends the model through the addition of a variable representing the effects of wealth on the baseline equation, as used in other studies such as Semmler and Zhang (2007). This modified Taylor rule is:

$$i_t^* = \mu + \lambda\pi_t + \gamma y_t + \delta w_t + \phi q_t \tag{1}$$

$$i_t = (1-\rho)i_t^* + \rho i_{t-1} + v_t \tag{2}$$

Where i_t^* is the target for the short-term nominal interest rate, π_t is the inflation rate, y_t is the output gap, defined as the percent deviation of actual real GDP from an estimate of its potential level, w_t is the asset price, q_t is the real exchange rate, i_t is the actual observable interest rate, ρ denotes the degree of interest rate smoothing and v_t is the error term also known as the interest rate smoothing shock.

Substituting (1) into (2) gives the following equation for the actual short-term interest rate:

$$i_t = (1-\rho)(\mu + \lambda\pi_t + \zeta y_t + \delta w_t + \phi q_t) + \rho i_{t-1} + v_t \tag{3}$$

Considering the US as the domestic country and equation (3) as the interest rate reaction function for the foreign country; the monetary policy reaction function for the US is the same as in equation (3) with $\phi = 0$.

To derive the Taylor rule based exchange rate equation, we follow the approach used by Molodtsova and Papell (2009). Letting ~ denote the foreign country variables, the interest rate differential is constructed by subtracting the Taylor rule equation for the foreign country from that of the domestic country, the US:

$$i_t - \tilde{\imath}_t = \alpha_0 + (\beta_{u\pi}\pi_t - \beta_{f\pi}\tilde{\pi}_t) + (\beta_{uy}y_t - \beta_{fy}\tilde{y}_t) + (\beta_{uw}w_t - \beta_{fw}\widetilde{w}_t) - \beta_q\tilde{q}_t + \rho_u i_{t-1} - \rho_f\tilde{\imath}_{t-1} + \eta_t, \tag{4}$$

where u and f are subscripts corresponding to the U.S. and the foreign country respectively, $\alpha_0 = \mu(1-\rho)$, $\beta_\pi = \lambda(1-\rho)$, $\beta_y = \zeta(1-\rho)$ and $\beta_w = \delta(1-\rho)$ for both countries and $\beta_q = \phi(1-\rho)$ for the foreign country.

Assuming that the UIP holds along with rational expectations:

$$E(\Delta s_{t+1}) = (i_t - \tilde{\imath}_t) \tag{5}$$

Substituting (4) into (5) produces the following Taylor rule exchange rate equation:

$$\Delta s_{t+1} = \alpha_0 + \beta_{u\pi}\pi_t - \beta_{f\pi}\tilde{\pi}_t + \beta_{uy}y_t - \beta_{fy}\tilde{y}_t + \beta_{uw}w_t - \beta_{fw}\widetilde{w}_t - \beta_q\tilde{q}_t + \rho_u i_{t-1} - \rho_f\tilde{\imath}_{t-1} + \eta_t \tag{6}$$

Where s_t is the natural log of the U.S. nominal exchange rate, defined as the US dollar per unit of foreign currency, so that an increase in s_t represents a depreciation of the US dollar. A further homogenous model was also estimated, where the two respective central banks are assumed to react identically to changes in inflation, the output gap, the wealth effect and that the interest rate smoothing coefficients are equal.[3] This in effect restricts the foreign coefficients to being equal to the domestic ones, so that $\beta_{u\pi} = \beta_{f\pi} \equiv \beta_\pi$, $\beta_{uy} = \beta_{fy} \equiv \beta_y$, $\beta_{uw} = \beta_{fw} \equiv \beta_w$ and $\rho_u = \rho_f \equiv \beta_i$. Therefore, the modified form of the Taylor rule model is as follows:

$$\Delta s_{t+1} = \boldsymbol{\alpha_0} + \boldsymbol{\beta_\pi(\pi_t - \widetilde{\pi}_t)} + \boldsymbol{\beta_y(y_t - \widetilde{y}_t)} + \boldsymbol{\beta_w(w_t - \widetilde{w}_t)} - \boldsymbol{\beta_q\widetilde{q}_t} + \boldsymbol{\beta_i(i_{t-1} - \tilde{\imath}_{t-1})} + \boldsymbol{\eta_t} \tag{7}$$

[3] Estimation and forecasting have been conducted for both homogenous and non-homogenous models, In general, we found the homogenous model generates better forecasts overall so we report these results.

The STR model for variable $\Delta \mathbf{s_{t+1}}$, has the following specification:

$$\Delta s_{t+1} = \boldsymbol{\phi_0} + \boldsymbol{\phi_1}' \, z_t + \left(\theta_0 + \boldsymbol{\theta_1}' \, \mathrm{z_t}\right) \cdot G(h_t;\ \gamma, c) + \varepsilon_t \tag{8}$$

where $\mathbf{z_t} = (\pi_t - \tilde{\pi}_t\, , y_t - \tilde{y}_t\, , w_t - \tilde{w}_t, \tilde{q}_t, i_{t-1} - \tilde{\imath}_{t-1})^{\grave{}}$, $\boldsymbol{\phi_1} = (\phi_\pi, \phi_y, \phi_w, \phi_q, \phi_i\,)^{\grave{}}$. $\boldsymbol{\theta_1} = (\theta_\pi, \theta_y,\ \theta_w, \theta_q, \theta_i)^{\grave{}}$. The error term $\boldsymbol{u_t}$ is assumed to be n.i.d. with zero mean and constant variance $\boldsymbol{\sigma^2}$, $\boldsymbol{G}$ is the transition function, $\boldsymbol{h_t}$ is the transition variable and $\boldsymbol{\gamma}$ is the transition parameter, also known as the speed of transition, which determines how quickly the transition between regimes occurs and is restricted by $\boldsymbol{\gamma > 0}$. $\boldsymbol{c}$ denotes a particular threshold level and corresponds to the value of the transition variable where the transition takes place. Both γ and c are estimated by the model.

Following Granger and Teräsvirta (1993), we use two alternative functional forms of the transition function in the context of the STR:

- Logistic Function:

$$G(h_t;\ \gamma, c) = \frac{1}{1 + \exp[-\gamma(h_t - c)]} \tag{9}$$

- Exponential Function:

$$G(h_t;\ \gamma, c) = 1 - exp[-\gamma(h_t - c)^2] \tag{10}$$

There are a number of potential sources of non-linearity in both the monetary policy and the exchange rate. For monetary policy they include nonlinearities in the Phillips Curve (Nobay and Peel, 2000). The justification for non-linearity in the exchange rate includes the effect of central bank intervention (Reitz *et al*., 2011), speculative restrictions (Baillie and Kilic, 2006) and heterogeneous trading behaviour (Sarantis, 1999).

The STR procedure (Teräsvirta 1994, Dijk *et al*. 2002) applied to this setting allows the transition to catch any smooth changes in our Taylor rule exchange rate model. The model assumes there are at least two regimes with different sets of coefficients and a transition variable which determines the movements across the regime.

Equation (8) combined with transition function (9) jointly define the logistic STR (LSTR) model and Equation (8) with transition function (10) forms an exponential STR (ESTR) model. Different functional forms of $\mathrm{G}(h_{\mathrm{t}};\ \gamma, \mathrm{c})$ correspond to different types of exchange rate switching behaviour.

For the LSTR model, the transition function is a monotonically increasing function of h_{t}. Therefore, the LSTR models describe relationships that change relative to the level of the threshold variable. Given that $\mathrm{G}(h_t;\ \gamma, \mathrm{c})$ is continuous and bounded between zero and one, the combined nonlinear coefficients $\phi_0 + \phi_1{}' + \theta_0 + \theta_1 \cdot \mathrm{G}(h_t;\ \gamma, \mathrm{c})$ will change monotonically from $\boldsymbol{\phi_0} + \phi_1{}'$ to $\left(\boldsymbol{\phi_0} + \phi_1{}' + \theta_0 + \theta_1\right)$ according to different values of h_{t}. When $h_{\mathrm{t}} - \mathrm{c} \rightarrow +\infty$, $\mathrm{G}(h_{\mathrm{t}};\ \gamma, \mathrm{c}) \rightarrow 1$ and coefficients become $\boldsymbol{\phi_0} + \phi_1{}' + \boldsymbol{\theta_0} + \theta_1$; when $h_{\mathrm{t}} - \mathrm{c} \rightarrow -\infty$, $\mathrm{G}(h_{\mathrm{t}};\ \gamma, \mathrm{c}) \rightarrow 0$ and the coefficients become $\boldsymbol{\phi_0} + \phi_1{}'$. In contrast to the logistic function, the exponential function is symmetric and U-shaped around c. It describes a form of dynamic behaviour which is the same for the high values of the transition variables as it is for low values.

The transition function $G(h;\, \gamma, c) \to 1$ both as $h_t - c \to -\infty$ and $h_t - c \to +\infty$ and the coefficient in this approach becomes $(\boldsymbol{\phi_0} + \phi_1{}' + \theta_0 + \theta_1)$. In the case of $h_t = c$, $G(h_t;\, \gamma, c) = 0$, the coefficients become $\boldsymbol{\phi_0} + \phi_1{}'$.

2.2 The Modelling Strategy

When testing for possible nonlinearity in the Taylor rule based exchange rate model, we use the procedure developed by Granger and Teräsvirta (1993) and Teräsvirta (1994). This modelling approach consists of three steps: specification, estimation and evaluation.

The Taylor rule STR model we study takes the following form:

$$\Delta s_{t+1} = \boldsymbol{\phi_0} + \boldsymbol{\phi_1}{}' \, z_t + \left(\boldsymbol{\theta_0} + \boldsymbol{\theta_1}{}' \, \mathbf{z_t}\right) \cdot G(\boldsymbol{h_t};\, \gamma, c) + \varepsilon_t \tag{11}$$

where $G(.)$ acts as the transition function; $\mathbf{z}_t = (\, \pi_t - \tilde{\pi}_t,\ y_t - \tilde{y}_t,\ \tilde{q}_t,\ i_{t-1} - \tilde{\imath}_{t-1},\ w_t - \tilde{w}_t)$ is the vector of regressors in the above models. The vector $\boldsymbol{\phi_1} = \left(\beta_\pi, \beta_y, \beta_i\, , \beta_w, \beta_q\right)$ and $\boldsymbol{\theta_1} = \left(\beta_\pi^*, \beta_y^*, \beta_i^*, \beta_w^*, \beta_q^*\right)$ contain the parameters from the linear and nonlinear parts of the model. We have chosen six different transition variables in the nonlinear estimation. These are the output gap, interest rate differential, inflation differential, wealth effect differential, real exchange rate and exchange rate volatility.[4]

[4] The nonlinearity in the Taylor rule exchange rate model may arise from either the Taylor rule part or the exchange rate. Therefore, the hypothesis of nonlinearity in the model can be tested simply by evaluating the setting in the functional form of the interest rate reaction function and exchange rate. The exchange rate volatility is used as a transition variable to study how the model is related to market risk by including a risk premium within the nonlinear part of the model.

After selecting the predetermined transition variable, we follow the approach proposed by Teräsvirta (1994), replacing the transition function $\boldsymbol{G}(\boldsymbol{h}_t;\ \boldsymbol{\gamma}, \boldsymbol{c})$ by a suitable Taylor series approximation in testing for the null of linearity against the alternative STR model. These tests are conducted through estimating the following auxiliary regression:

$$\Delta s_{t+1} = \delta_0' \boldsymbol{z}_t + \sum_{j=1}^{3} \delta_j' \tilde{\boldsymbol{z}}_t h_t^j + \varepsilon_t^* \tag{12}$$

Where $\tilde{z}_t$ is the vector of variables in z_t without the constant; h_t is one of the elements of z_t. The null hypothesis is of linearity (H_0): $\delta_1 = \delta_2 = \delta_3 = 0$; whilst the alternative hypothesis is at least one of $\delta_j \neq 0,\ j = 1,2,3$. As suggested by Teräsvirta (1994), F-versions of the LM test statistics are employed as these have better size properties than the χ^2-statistic.[5]

Once the null hypothesis of linearity is rejected in favour of STR nonlinearity, we can choose the appropriate form of the transition function. The decision is based on testing the order of the polynomial in the auxiliary regression (12). Granger and Teräsvirta (1993) and Teräsvirta (1994) proposed the following sequence of null hypotheses:

$$H_{03}:\ \delta_3 = 0 \tag{13}$$

$$H_{02}:\ \delta_2 = 0\ given\ \delta_3 = 0 \tag{14}$$

$$H_{01}:\ \delta_1 = 0\ given\ \delta_3 = 0, \delta_2 = 0 \tag{15}$$

According to Teräsvirta (1994), the decision rules for choosing between LSTR and ESTR models are the following: We compare the *p*-value of the three F-tests, if the p-value of the test

[5] In small or moderate sized samples, the χ^2-statistic may be heavily oversized (Dijk *et al.* (2002)).

corresponding to H_{02} is the smallest among the three, then we select the ESTR model; otherwise a LSTR model is chosen.

Both the LSTR and ESTR model are estimated using NLLS with a grid search for the parameters γ and c with respect to the nonlinear optimization, in this case the result yielding the minimum RSS. For the γ estimation, we scale the transition function, by dividing it using the standard deviation of h_t (i.e. $\hat{\sigma}_s$) for the LSTR models and by the variance of h_t (i.e. ${\hat{\sigma}_s}^2$) in the case of ESTR. The transition function is standardised to make it easier to compare the estimates of the transition parameters across different equations.[6]

For the purpose of assessing the statistical adequacy of the STR models in this study we follow Eitrheim and Teräsvirta (1996) who propose an LM test for the null hypothesis of no error autocorrelation and LM-type tests for the null of no remaining nonlinearity and that of parameter constancy. Following Sarantis (1999) we also run the Jarque-Bera test for normality in the errors, as well as a test for ARCH effects.

3. Results and Discussion

3.1 Data

Throughout our empirical investigation we use quarterly data which consists of the exchange rate returns measured in log-differences, and the Taylor rule based economic fundamentals for the United Stated, the United Kingdom, Sweden and Australia. The variables are π_t the inflation

[6] This is also recommended by Granger and Teräsvirta (1993) and Teräsvirta (1994). They have argued that scaling the transition variable by its own standard deviation before empirical estimation not only speeds up the convergence but also improves the stability of the nonlinear least squares estimation algorithm.

rate, y_t the output gap, defined as the percent deviation of actual real GDP from an estimate of its potential level, w_t is the asset price[7], q_t is the real exchange rate and i_t is the actual observable interest rate. The United Stated, the United Kingdom, Sweden and Australia are selected because of the availability of the wealth data, especially for housing and also as a result of their strong housing markets. Due to data availability, the time period for these countries differs depending on the measure of the wealth effect. When stock prices represent the wealth effect, the data ranges from 1979Q1 to 2008Q4, whereas when house prices are used, the data is from 1980Q1 to 2008Q4 (data from 1975 to 1979 is used to generate the output gap). As a measure of wealth, stock prices have been chosen to represent the wealth effect for the UK and Australia[8]. This is because they have particularly strong equity markets, whereas in Sweden they are of less importance compared to the banking system, therefore house prices were found to be the most appropriate wealth effect for Sweden.

All variables, except interest rates, are expressed in logarithms. The inflation rate is the annual inflation rate calculated using the CPI over the previous 4 quarters and real GDP is used to measure the level of output. As in other studies, the output gap is constructed as the percentage change of actual output from a quadratic trend, using an expanding window as in Molodtsova *et al*. (2008), in addition the wealth effect is measured as a wealth gap, constructed in the same way as the output gap using the quadratic trend. Therefore for the first vintage 1979:Q1, the trend is

[7] Stock prices and house prices are used as a proxy for wealth and have been employed to analyse the wealth effect in the context of exchange rate models. The data are measured as deviation of natural log of stock price or house price from a quadratic trend.

[8] This is based on the linear estimation of the model, in which stock prices were found to be the most significant determinant for these two countries. Results available on request.

calculated using data from 1975:Q1 to 1978:Q4. For each subsequent vintage, we update the trend by one quarter. The real foreign/U.S. exchange rate is calculated as the percentage deviation of the nominal exchange rate from the target defined by PPP (i.e., $\tilde{q}_t = s_t - (p_t - p_t^*)$, where p_t and p_t^* are natural logarithms for U.S. and foreign price levels, respectively, as measured by respective CPI levels). Money market rates are used as a measure of short-term interest rates. The nominal exchange rate is defined as the U.S. dollar price of foreign currency and is taken as the end-of-month exchange rate and as in Ince (2014).

Studies by Molodtsova et al. (2008), among others, have highlighted the importance of real-time data for the purpose of Taylor rule-based exchange rate forecasting. Real-time data are based on vintages of data that are available to researchers at each point in the time that the forecasting exercise is run (i.e., before data revisions are applied). The real time output data was collected from the OECD Real-Time Data and Revisions Dataset and the Real Time Dataset for the OECD – Dallas Fed[9]. Exchange rate volatility is measured using the conditional volatility series produced from a GARCH (1, 1) model which has been widely used to proxy the risk premium. All the data, other than the real time data were obtained from Thomson DataStream and the International Monetary Fund International Statistics (IMF). The quarterly closing prices of the main stock market indexes are used to represent equity prices in each country. House price indices are taken from Oxford Economics and measure the quarterly house price.

In addition a further test on the UK/US model was conducted using the data up to 2015. This included using the shadow policy rate of Wu and Xia (2016)[10], the use of this variable overcomes

[9] The data can be found at: http://stats.oecd.org/mei/default.asp?rev=1

[10] We would like to thank a referee for suggesting the use of this dataset. The data is available at https://sites.google.com/site/jingcynthiawu/home/wu-xia-shadow-rates.

the problem of the near zero bound policy rate after 2008. This enables us to assess whether there is evidence of non-linearity over a time period including the recent financial crisis and whether the model is able to outperform other models during the crisis period. However, Table 2 suggests that there is little evidence of non-linearity, as the only case in which non-linearity is found involves the use of volatility as the transition variable. We note that there is evidence of autocorrelation in this test[11]. The plots of all data used in this study appear in Figure 1.

3.2 Nonlinear estimation results

Table 1 contains the p-values of the LM tests for the full sample (we use the whole sample for this test) as the estimation of the model and subsequent forecasting is done using a non-linear approach. The first column reports the results of the tests for linearity against non-linearity based on the STR models for each transition variable considered[12] (i.e. H_0). The following columns show the results of the model selection tests, which determine whether we use the LSTR or the ESTR approach (i.e. H_{01}, H_{02}, H_{03}) and the subsequent non-linear model specification. According to Teräsvirta (1994), since it is possible for the three hypotheses (H_{01}, H_{02}, H_{03}) to be simultaneously rejected, we have selected the one with the strongest level of rejection (i.e. lowest p-value). The results provide evidence in favour of a nonlinear specification for the Taylor rule based exchange rate model, although the result is sensitive to the transition variable. We proceed to use the specifications where we find evidence of non-linearity and in addition pass

---

[11]Despite evidence of autocorrelation, the forecasts from this longer dataset are included with the results, showing that it outperforms the benchmark forecast models. .

[12] All variables were initially tested for non-stationarity using the Ng and Perron (2001) test, which suggested all were stationary at the 5% level of significance. Also these tests exclude the dummy variables, when the dummy variables were added it made little difference to the results so are not reported.

the diagnostic tests prior to considering the model for forecasting. By excluding the specifications that failed the diagnostic tests, we are able to reduce the number of models for forecasting. For each country, there is evidence of nonlinearity based on the transition variables considered. As mentioned in the STR literature, the final decision on this can be postponed to the evaluation stage of the modelling strategy as in Teräsvirta (1994, 1996) and Dijk et al. (2002). In this study, we will follow the recommendation of Teräsvirta (1994). To be selected as a model for the out-of-sample forecasting, the model needed to provide evidence of non-linearity and pass the diagnostic tests. The decision regarding the best performing model will then be made based on the model evaluation and forecasting performance.

We have conducted a number of diagnostic tests on the non-linear LSTR models, in order to verify their statistical adequacy. We run the Jarque-Bera normality test for the residuals. Following Eitrheim and Teräsvirta (1996), the Breusch-Godfrey LM test has been used to test for first and fourth order autocorrelation in the errors. This includes the LM (1) and LM (4) tests in which the null is of no first and up to forth order autocorrelation. To test for ARCH effects we run the ARCH-LM (1) test in which the null is no first order residual ARCH effect. The test for no remaining nonlinearity examines whether there is any remaining nonlinearity in the model after the initial non-linearity has been controlled for using the STR specifications described above. For parameter constancy, we again follow Eitrheim and Teräsvirta (1996), whose approach is to test the null of parameter constancy against three alternative hypotheses: $H1$: the parameters change monotonically over time; $H2$: that the change is symmetric with respect to an unknown point in time; $H3$, change is possibly non monotonic but not necessarily symmetric. Rejection of either one of the null hypotheses $H1$, $H2$ and $H3$ indicates parameter non-constancy, otherwise the parameters are time-invariant. Tables 3 and 4 suggest that for all three countries the diagnostic tests are predominantly passed, although in a couple of cases for all the countries there is evidence of higher order autocorrelation, including the test for the UK using the whole data sample and for the restricted data set using the interest rate as the transition variable. Due to the sensitivity of the estimation of these models to the presence of autocorrelation, we have proceeded to the out-of-sample forecasts in those cases where the diagnostic tests were passed.

3.3 ***Modelling the Transition***

Based on the findings of non-linearity with certain transition variables, Figure 2 shows plots of the transition functions over time during our sample period. The change in the parameters, which depend on different transition variables, can be viewed as an indicator of the overall economic conditions or the monetary policy stance. To save space we have only reported the interest rate differential plots which are common to Sweden and Australia, and the volatility plots for the UK, as this allows us to demonstrate the results from the LSTAR model as well as ESTAR models. For the UK, the first evidence of any transition is during the early 1980s, reflecting an era of relatively volatile exchange rates, with a move to zero around 1992 during the ERM crisis, when the UK was forced out. From this Figure, it is interesting to note that the transition functions between 1994 and 2008 are mostly close to unity, representing a period of relative stability for the UK currency.

For Sweden, the main change in parameters and the frequent shift of transition functions ends around 1994. This pattern reflects Swedish economic policy at the time, which experienced substantial interventions in the foreign exchange markets during the fixed rate regime period (i.e. before 1990). Following the banking crisis, there was a policy realignment within the Swedish economy in early 1990. It is also noticeable that between 1990 and 1994, the transition functions attained values mostly in the upper regime with values close to unity, this reflects the severe banking and subsequent financial and economic crises experienced by Sweden in the early 90s.

Similar to the plots of the UK and Sweden, the estimated transition function for Australia frequent shifts between regimes and large changes in parameters have occurred mostly before Australia adopted floating exchange rates at the end of 1983.

Overall, the nonlinear specification improves upon the linear one by explaining some of the variation in the exchange rate related to the extreme peaks of various transition variables. In some respects the results follow the study of Bruggemann and Riedel (2011) on interest rate reaction functions in which they find that non-linearity tends to set in when the economy goes into a recessionary time period, often in conjunction with a financial crisis, especially for the UK. Another potential reason for the finding of non-linearity is that the Taylor rule tends to hold when the inflation rate is above a certain threshold value. The transition functions over time indicate that transition between regimes with large changes in parameters occurred most frequently prior to the introduction of the floating exchange rate system and inflation targeting. This is because the nonlinear Taylor type exchange rate relationships are based on Taylor rule interest rate models which are mainly used in studying the change and setting of monetary policy. Figure 3 presents the estimated transition function of the LSTR and ESTR models, as before the interest differential is used for Sweden and Australia, whilst volatility is used for the UK, this demonstrates the number of observations above and below the threshold. These figures provide evidence of strong nonlinear behaviour for these models and give supportive evidence for the smooth change between two extreme regimes in most of the cases, especially with the ESTR models. For the LSTR model, when volatility is the transition variable, the estimated threshold is above the halfway point between the regimes. Therefore, almost all of the observations belong to the left hand tail of the transition function as is seen from the figure. The value of the transition function has remained close to zero for most of the volatility values. Thus a linear model could do almost as good a job as the LSTR model. However, since the LSTR model had a better fit and there is evidence of nonlinearity shown in both the figure and the test, the result is reported here.

3.4 Out-of-sample forecasting

The non-linear STR models are now analysed in terms of their out-of-sample forecasting performance, including all the transition variables which produced evidence of non-linearity and passed all the diagnostic tests as well as the longer dataset using the shadow interest rate (volatility15). This is done by comparing each nonlinear specification's forecasting performance with those of the equivalent linear model, a non-linear uncovered interest rate parity based model (UIP) as well as the random walk, the standard benchmark for forecasting exchange rates. The UIP model has been adopted as a comparison as it is a standard exchange rate model, which has previously been modelled using non-linear approaches, as noted by Baillie and Kilic (2006). Firstly, all models were re-estimated up to 1999Q4 and these estimates were used to generate a set of rolling forecasts for 2000Q1 to 2008Q4. Each out-of-sample forecast is constructed using all the data up to the forecasting period. So, in total, we obtain 36 forecasts for each model.

The mean square prediction error (MSPE) is adopted as the measure of the forecast performance of these models as it is the most commonly used criterion for deciding on which one from a set of models has the best forecasting accuracy. For non-nested models, a commonly used test of significance is the Diebold and Mariano (1995) and West (1996) MSE-t test (DMW test), as well as the McCracken (2007) out-of-sample F-type test of equal MSE. However, because the Taylor rule exchange rate model used here is a nested model, the test properties are likely to be different[13]. In the case of nested models, a number of forecast performance evaluation criteria have become popular, such as the Clark and West (2006) (henceforth, CW) test, the Clark and McCracken's (2001) encompassing test, the modified Diebold and Mariano (1995) encompassing test, as suggested by Harvey et al. (1998)[14] and the fluctuation test developed by Giacomini and Rossi (2010). The latter test evaluates the fluctuations in the relative predictive abilities of the models throughout the span of the data. This test can be viewed as a plot of the standardized sample path of the relative measure of the local performance (difference in MSFEs), with the respective critical values. When the critical value schedule is crossed it suggests the model outperforms the competitor at this specific time point.

Tables 6 and 7 present the results for the out-of-sample forecasts, based on the transition variables that produce evidence of non-linearity and pass the diagnostic tests. As is evident from these results the non-linear Taylor rule based model outperforms all the alternative specifications considered, including with the longer dataset using the shadow interest rate. The MSPEs are below unity in every specification, showing that for the countries studied, the non-linear STR model is producing more accurate forecasts than the equivalent linear models, UIP model and random walk. The subsequent columns contain the test statistics described above, which measure forecast performance, where the benchmark model is nested. The CW statistics with respect to both benchmarks, provide evidence that the null hypothesis of equal forecasting accuracy can be

[13] According to Clark and McCracken (2001) and a further study by McCracken (2007) these statistics are not distributed normally for two forecasts from a nested model. In addition Clark and McCracken (2012) show the distributions of the MSE-t and MSE-F statistics are non-standard when models are nested. This means that using standard normal critical values results in poorly sized tests.

[14] The DMW and CW tests do not necessarily lead to the same result. The CW tests for the regression coefficient being zero instead of if the sample MSPE from the model-based forecast is less than the sample MSPE from the benchmark forecast. The non-nested test results are available from the authors on request.

rejected. When the benchmark is the random walk, we notice that the STR specifications outperform the random walk model in almost all cases at the 5% significant level. This result for the UK/US exchange rate is similar to Pavlidis *et al.* (2012), who find that the non-linear models outperform the equivalent linear models, although they used a different approach. When comparing the forecasting performance of the STR specifications with the linear models and UIP, Overall we find that for every specification where there is evidence of nonlinearity, the non-linear model usually outperforms the linear model in terms of forecasting accuracy at the 5% significance level .The only cases in which the non-linear model doesn't significantly outperform the linear model is with the Australian data. It therefore appears that the out-of-sample forecasting performance of the STR specifications is more accurate than both the equivalent linear specifications, a UIP model and the random walk.

Finally, Figures 4 present the Giacomini and Rossi (2010) fluctuation test results from the forecasts of the exchange rates over all three currencies and the relevant transition variables. Overall the results from these Taylor rule based exchange rate models do not outperform the linear models over the entire range of the sample considered, although the failure is limited to a number of specific short time periods, where the currencies experienced excessive volatility, such as the time period at the end of the sample when the 2008 financial crisis began. In addition there are some country specific events that affected the forecast performance, such as the banking crisis in Sweden in the early 1990s and the collapse in commodity prices in the late 1990s which affected the Australian currency.

4 Conclusion

This study analyses the forecasting performance of the non-linear Taylor rule based exchange rate model, complementing recent research which has found that the linear version of this model outperforms a random walk. Using quarterly data on dollar-sterling, dollar-Swedish krona and dollar-Australian exchange rates over the period 1979 to 2008, we find evidence of nonlinearities in the exchange rate with respect to several macroeconomic determinants, suggesting that the Taylor rule exchange rate models can be improved in some cases by considering regime changes.

The presence and form of the non-linearity and the transition process appears to vary across the countries studied, reflecting the differences in these economies. In general, the interest rate differential has been found to be an important source of nonlinearities in exchange rates for all the countries studied. For both Sweden and Australia, the ESTR model with interest rate differences as the transition variable delivers a well specified model, which prevails over other nonlinear models. For the UK's exchange rate, the estimation results based on the output gap and volatility as the transition variable generally give the best specification. Comparing to the benchmarks of the random walk and the linear Taylor rule models the STR models appear to have better out-of-sample predictive performance which can be viewed as strong evidence in favour of the use of the non-linear Taylor rule model in this field of economic research.

Given the importance of predicting and explaining exchange rate movements for monetary policy, the main implication of the study is that using a non-linear approach to modelling and forecasting can produce better outcomes than the more conventional linear approaches, suggesting when forecasting exchange rates non-linear models need to be used in conjunction with the linear approach, depending on economic conditions and specific countries. In addition the inclusion of wealth effects within the model adds to the previous evidence showing that possibly as a result of increased capital movements between international financial markets, asset prices have an important effect on exchange rates. Future research in this area could consider alternative non-linear approaches to modelling and a wider selection of potential transition variables

References

Baillie, R. T. and Kiliç, R. (2006). Asymmetry and nonlinearity in uncovered interest rate parity. *Journal of International Money and Finance* 25(2). 22-47.

Boero, G. and Marrocu, E. (2002). The performance of non-linear exchange rate models: a forecasting comparison, *Journal of Forecasting*, 21, 513-542.

Bollerslev, T. (1990). Modelling the coherence in short-run nominal exchange rates: a multivariate generalized ARCH model, *The Review of Economics and Statistics*, 498-505.

Brüggemann, R., and Riedel, J. (2011). Nonlinear interest rate reaction functions for the UK. *Economic Modelling*, 28(3), 1174-1185.

Case, K. E., Quigley, J. M., & Shiller, R. J. (2005). Comparing wealth effects: the stock market versus the housing market. *Advances in macroeconomics*, 5(1).

Cheung, Y.-W., Chinn, M.D. and Pascual, A.G. (2005). Empirical exchange rate models of the nineties: are any fit to survive? *Journal of International Money and Finance,* 24: 1150-1175.

Clarida, R., Galí, J. and Gertler, M. (1998). Monetary policy rules in practice: Some international evidence. *European Economic Review* 42(6): 1033-1067.

Clark, T. E., and McCracken, M. W. (2001). Tests of equal forecast accuracy and encompassing for nested models. *Journal of Econometrics, 105*(1), 85-110.

Clark, T. E., and McCracken, M. W. (2012). Reality checks and comparisons of nested predictive models. *Journal of Business & Economic Statistics, 30*(1), 53-66.

Clark, T. E., & West, K. D. (2006). Using out-of-sample mean squared prediction errors to test the martingale difference hypothesis. *Journal of Econometrics*, 135(1-2), 155-186.

Clark, T., and West, K. (2007). Approximately normal tests for equal predictive accuracy in nested models. *Journal of Econometrics, 138*(1), 291-311.

De Grauwe, P. and Grimaldi, M. (2005). The exchange rate and its fundamentals in a complex world, *Review of International Economics* 13, 549-575

Diebold, F., and Mariano, R. (1995). Comparing Predictive Accuracy. *Journal of Business & Economic Statistics, 13*, 253-263.

Dijk, D. v., Teräsvirta, T. and Franses, P. (2002). Smooth transition autoregressive models—a survey of recent developments. *Econometric Reviews* 21(1): 1-47.

Eitrheim, Ø., & Teräsvirta, T. (1996). Testing the adequacy of smooth transition autoregressive models. *Journal of Econometrics*, 74(1), 59-75.

Enders, W. and Pascalau, R. (2015). Pretesting for multi-step-ahead exchange rate forecasts for STAR models, *International Journal of Forecasting*, 31, 473-487.

Engel, C. (1994). Can the Markov switching model forecast exchange rates? *Journal of International Economics*, 36(1-2), 151-165.

Giacomini, R., and Rossi, B. (2010). Forecast comparisons in unstable environments. *Journal of Applied Econometrics, 25*(4), 595-620.

Granger, C. W. and Teräsvirta , T. (1993). Modelling non-linear economic relationships. Oxford University Press, Oxford.

Granger, C., Huangb, J. and Yang, C. (2000). A bivariate causality between stock prices and exchange rates: evidence from recent Asianflu. *The Quarterly Review of Economics and Finance* 40(3), 337-354.

Harvey, D. S., Leybourne, S. J., and Newbold, P. (1998). Tests for forecast encompassing. *Journal of Business & Economic Statistics, 16*(2), 254-259.

Ince, O. (2014). Forecasting Exchange Rates Out-of-Sample with Panel Methods and Real-Time Data. *Journal of International Money and Finance*, 43, 1-18.

Lutkepohl, H, Teräsvirta, T. and Wolters, J. (1999). Investigating stability and linearity of a German M1 money demand function. *Journal of Applied Econometrics*, 14, 511-525.

Liu, Y., Nikolsko-Rzhevskyy, A., and Prodan, R. (2010). The comparative performance of alternative out-of-sample predictability tests with non-linear models. University of Houston Working Paper.

McCracken, M. W. (2007). Asymptotics for out of sample tests of Granger causality. *Journal of Econometrics, 140*(2), 719-752.

Ng, S., & Perron, P. (2001). Lag length selection and the construction of unit root tests with good size and power. *Econometrica*, 69(6), 1519-1554.

Michael, P., Nobay, A. and Peel, D. (1997). Transactions costs and nonlinear adjustment in real exchange rates; An empirical investigation. *Journal of Political Economy* 105(4), 862-879.

Molodtsova, T., Nikolsko-Rzhevskyy, A., and Papell, D. (2008). Taylor rules with real-time data: A tale of two countries and one exchange rate. *Journal of Monetary Economics, 55*, S63-S79.

Molodtsova, T. and Papell, D. (2009). Out-of-sample exchange rate predictability with Taylor rule fuundamentals, *Journal of International Economics*, 77, 167-180.

Nobay, A. and Peel, D. (2000). Optimal monetary policy with a non-linear Phillips curve, *Economics Letters*, 67, 159-164.

Pavlidis, E., Paya, I. and Peel, D. (2012). Forecast evaluation of non-linear models: The case of long-span real exchange rates, *Journal of Forecasting*, 31, 580-595.

Reitz, S., Ruelke, J. C., and Taylor, M. (2011). On the Nonlinear Influence of Reserve Bank of Australia Interventions on Exchange Rates. *Economic Record,* 87, 465-479.

Qin, T. and Enders, W. (2008). In sample and out-of-sample properties of linear and non-linear Taylor rules, *Journal of Macroeconomics,* 30, 428-443.

Rapach, D. E., and Strauss, J. K. (2008). Structural breaks and GARCH models of exchange rate volatility, *Journal of Applied Econometrics*, 23(1), 65-90.

Rapach, D. and Wohar, M. (2006). The out-of-sample forecasting performance of non-linear models of real exchnage rate behaviour.*International Journal of Forecasting*, 22, 341-361.

Sarantis, N. (1999). Modelling non-linearities in real effective exchnage rates, *Journal of International Money and Finance*, 18, 27-45.

Semmler, W. and Zhang, W. (2007). Asset price volatility and monetary policy rules: a dynamic model and empirical evidence. *Economic Modelling,* 24(3), 411-430.

Taylor, J. B. (1993). Discretion versus policy rules in practice. In *Carnegie-Rochester conference series on public policy* (Vol. 39, pp. 195-214). North-Holland.

Teräsvirta, T. (1994). Specification, estimation, and evaluation of smooth transition autoregressive models. *Journal of the American Statistical Association*, 89, 208-218.

Teräsvirta, T. (2006). Forecasting economic variables with nonlinear models. *Handbook of Economic Forecasting* 1, 413-457.

Teräsvirta , T. and Anderson, H. (1992). Characterizing nonlinearities in business cycles using smooth transition autoregressive models. *Journal of Applied Econometrics* 7(1), S119-S136.

Wang, R, Morley, B. and Ordonez, J. (2016). The Taylor rule, wealth effects and the exchnage rate. *Review of International Economics*, 24, 282-301.

West, K. (1996). Asymptotic inference about predictive ability. *Econometrica*, 1067-1084.

Wu, J. and Xia, F. (2016). Measuring the macroeconomic impact of monetary policy at the zero lower bound", *Journal of Money, Credit, and Banking*, 48(2-3), 253-291.

Table 1 linearity tests on the Taylor rule model using data 1980Q1 -2008Q4

	Transition variable	H_0	H_{01}	H_{02}	H_{03}	*Type of model*
UK	$\pi_t - \tilde{\pi}_t$	0.4604	0.1971	0.6014	0.2564	Linear
	$y_t - \tilde{y}_t$	**0.0684**	0.2012	**0.0252**	0.3290	ESTR
	$i_{t-1} - \tilde{i}_{t-1}$	**0.0476**	0.1726	**0.0922**	0.1009	ESTR
	$w_t(s) - \tilde{w}_t(s)$	0.8035	0.9047	0.9463	0.3011	Linear
	$w_t(h) - \tilde{w}_t(h)$	0.6793	0.7974	0.8248	0.3151	Linear
	$\tilde{q}_t$	0.2130	0.2814	0.3063	0.1920	Linear
	volatility	**0.0115**	**0.0629**	0.1292	**0.0122**	LSTR
Sweden	$\pi_t - \tilde{\pi}_t$	**0.0025**	**0.4906**	**0.0406**	**0.0048**	LSTR
	$y_t - \tilde{y}_t$	**0.0966**	**0.0692**	0.1328	0.1767	LSTR
	$i_{t-1} - \tilde{i}_{t-1}$	**0.0000**	**0.0677**	**0.0000**	0.1204	ESTR
	$w_t(s) - \tilde{w}_t(s)$	0.9078	0.8308	0.9358	0.6467	Linear
	$w_t(h) - \tilde{w}_t(h)$	**0.0212**	0.1815	0.1276	**0.0193**	LSTR
	$\tilde{q}_t$	**0.0001**	**0.0651**	**0.0040**	**0.0006**	LSTR
	volatility	**0.0000**	**0.0134**	**0.0071**	**0.0005**	LSTR
Australia	$\pi_t - \tilde{\pi}_t$	**0.0017**	**0.0028**	**0.0063**	**0.0311**	LSTR
	$y_t - \tilde{y}_t$	**0.0001**	**0.0041**	**0.0001**	**0.0458**	ESTR
	$i_{t-1} - \tilde{i}_{t-1}$	**0.0124**	0.1436	**0.0020**	**0.3039**	ESTR
	$w_t(s) - \tilde{w}_t(s)$	0.8540	0.2432	0.6367	0.9234	Linear
	$w_t(h) - \tilde{w}_t(h)$	**0.0026**	**0.0023**	**0.0014**	0.1669	ESTR
	$\tilde{q}_t$	0.1157	0.6482	0.7459	0.1090	Linear
	volatility	0.4839	0.2983	0.5433	0.3531	Linear

Notes: the table presents p-values of the linearity test after introducing dummy variables in the models for which the null hypothesis of linearity is tested against the alternative of the STR model; bold values correspond to rejection of the null at the 10% level. When the H_0 is rejected against a certain alternative hypothesis, we proceed with the estimation of the corresponding STR model. In cases that the H_0 is rejected against more than one of the alternative hypotheses considered (H_{01}, H_{02}, and H_{03}) we proceed to the estimation of the STR model corresponding to the alternative hypothesis for which the p-value is the lowest.

Table 2 Linearity test on the Taylor rule model using data 1980Q1 -2015Q4

	Transition variable	H_0	H_{01}	H_{02}	H_{03}	*Type of model*
UK	$\pi_t - \tilde{\pi}_t$	0.2830	**0.0351**	0.1355	0.7463	Linear
	$y_t - \tilde{y}_t$	0.3006	0.4972	0.4511	0.1858	Linear
	$i_{t-1} - \tilde{i}_{t-1}$	0.1567	0.0841	0.3265	0.1008	Linear
	$w_t(s) - \tilde{w}_t(s)$	0.4389	0.9935	0.6774	0.1730	Linear
	$w_t(h) - \tilde{w}_t(h)$	0.7609	0.9315	0.7240	0.5787	Linear
	$\tilde{q}_t$	0.5077	0.5877	0.3419	0.7091	Linear
	volatility	**0.0315**	0.1925	0.1362	**0.0506**	LSTR

Notes: The shadow policy rate of Wu and Xia (2016) is used here which allows for the estimation of the UK/US model using quarterly data from 1980 to 2015.

Table 3. Diagnostic results for the nonlinear Taylor rule model

	Panel A: UK				Panel B: Sweden					
Model	*ESTR*	*ESTR*	*LSTR*	*LSTR*	*LSTR*	*LSTR*	*ESTR*	*LSTR*	*LSTR*	*LSTR*
Transition variable (s_t)	$y_t - \tilde{y}_t$	$i_{t-1} - \tilde{i}_{t-1}$	*Volatility (-15Q4)*	*Volatility (-08Q4)*	$\pi_t - \tilde{\pi}_t$	$y_t - \tilde{y}_t$	$i_{t-1} - \tilde{i}_{t-1}$	$w_t - \tilde{w}_t(h)$	$\tilde{q}_t$	*volatility*
Residual Tests										
JB	0.241	0.893	0.732	0.441	0.843	0.604	0.427	0.234	0.941	0.848
ARCH-LM(1)	0.311	0.436	0.190	0.235	0.710	0.737	0.785	0.507	0.669	0.823
LM(1)	0.539	0.215	0.152	0.245	0.298	0.370	0.196	0.040**	0.525	0.103*
LM(4)	0.075*	0.002**	0.004**	0.121	0.475	0.491	0.261	0.075*	0.702	0.079*
Remaining Nonlinearity										
$\pi_t - \tilde{\pi}_t$	0.9673	0.8992	0.4060	0.8749	0.901	0.552	0.968	0.958	0.941	0.969
$y_t - \tilde{y}_t$	0.5169	0.5982	0.7457	0.3917	0.804	0.891	0.091*	0.170	0.274	0.294
$i_{t-1} - \tilde{i}_{t-1}$	0.7177	0.9758	0.5391	0.9546	0.211	0.216	0.242	0.271	0.777	0.871
$w_t - \tilde{w}_t(s)$	0.9892	0.9171	0.6703	0.9283	0.264	0.850	0.252	0.656	0.994	0.985
$\tilde{q}_t$	0.2750	0.7606	0.6702	0.8182	0.685	0.510	0.264	0.513	0.559	0.920
volatility	0.3407	0.8215	0.2743	0.6586	0.375	0.022	0.470	0.172	0.062*	0.474
Parameter Constancy										
H_1	0.316	0.369	0.661	0.919	0.418	0.999	0.228	0.103	0.348	0.154
H_2	0.517	0.613	0.317	0.331	0.498	0.958	0.782	0.864	0.026**	0.546
H_3	0.402	0.995	0.340	0.734	0.090*	0.063*	0.148	0.018**	0.687	0.751

Notes: numbers in this table are p-values. ** and * represent rejection of the null at the 5% and 10% significance levels, respectively. JB denotes the Jarque-Bera test for the null of the normality of residuals. The Breusch-Godfrey LM test, is used to test for serial correlation in the errors. LM (1) and LM (4) denote the null of no first and forth order serial correlation. ARCH-LM (1) denotes the null of no first order residual heteroskedasticity. The test for no remaining linearity examines whether there exists some remaining nonlinearity in the process after the initial non-linearity has been controlled for. The parameter constancy test is the version proposed by Eitrheim and Teräsvirta (1996), in the context of which the null of parameter constancy is tested against three alternative hypotheses: $\boldsymbol{H_1}$: the parameters change monotonically over time; $\boldsymbol{H_2}$: that the change is symmetric with respect to an unknown point in time; $\boldsymbol{H_3}$, change is possibly non monotonic but not necessarily symmetric.

Table 4. Diagnostic results for the nonlinear Taylor rule model – Australia

Model	*ESTR*	*ESTR*	*LSTR*	*ESTR*
Transition variable (s_t)	$i_{t-1} - \tilde{i}_{t-1}$	$w_t - \widetilde{w}_t(h)$	$\pi_t - \widetilde{\pi}_t$	$y_t - \widetilde{y}_t$
Residual Tests				
JB	0.659	0.503	0.522	0.662
ARCH-LM(1)	0.675	0.708	0.897	0.984
LM(1)	0.216	0.161	0.301	0.196
LM(4)	0.346	0.150	0.024*	0.211
Remaining Nonlinearity				
$\pi_t - \widetilde{\pi}_t$	0.962	0.962	0.964	0.442
$y_t - \widetilde{y}_t$	0.225	0.878	0.547	0.964
$i_{t-1} - \tilde{i}_{t-1}$	0.821	0.943	0.985	0.983
$w_t - \widetilde{w}_t(s)$	0.954	0.165	0.820	0.957
$\widetilde{q}_t$	0.680	0.860	0.656	0.751
volatility	0.796	0.062	0.746	0.598
Parameter Constancy				
H_1	0.779	0.059	0.632	0.751
H_2	0.100	0.144	0.138	0.159
H_3	0.116	0.530	0.214	0.746

Note: see table 3 notes.

Table 5. Diagnostic results for the nonlinear URIP model with $i_{t-1} - \tilde{i}_{t-1}$ as transition variable

	UK	*UK*	*Sweden*	*Australia*
Sample	*80Q1:08Q4*	*80Q1:15Q4*	*80Q1:08Q4*	*80Q1:08Q4*
Model	*LSTR*	*LSTR*	*LSTR*	*ESTR*
Residual Tests				
JB	0.808	0.072*	0.126	0.613
ARCH-LM(1)	0.157	0.137	0.635	0.949
LM(1)	0.696	0.056*	0.832	0.880
LM(4)	0.062*	0.011**	0.628	0.176
Remaining Nonlinearity				
$i_{t-1} - \tilde{i}_{t-1}$	0.969	0.926	0.665	0.948
Parameter Constancy				
H_1	0.465	0.927	0.583	0.254
H_2	0.494	0.906	0.576	0.228
H_3	0.425	0.737	0.584	0.188

Note: see table 3 notes.

Table. 6. Forecasting results

		Panel A: Random Walk						Linear Model (adjusted Taylor rule)					
Country	***Transition variable***	***MSPE***	***CW***	**ENC-F**	**ENC-t**	**MSE-F**	**MSE-t**	***Theil's U***	***CW***	**ENC-F**	**ENC-t**	**MSE-F**	**MSE-t**
UK	$y_t - \tilde{y}_t$	0.002	2.811**	62.149**	4.313**	8.760**	0.577**	0.628	2.351**	3.349*	1.184*	0.226*	0.044*
	volatility	0.002	3.554**	134.593**	5.025**	75.388**	2.515**	0.512	2.247**	21.594**	3.100**	20.500**	1.826**
	volatility15	0.002	2.499**	74.128**	4.622**	7.111**	0.328**	0.518	4.120**	65.859**	4.408**	12.984**	0.600**
Sweden	$i_{t-1} - \tilde{i}_{t-1}$	0.004	2.717**	143.647**	3.438**	89.353**	1.932**	0.479	3.599**	5.684**	1.952**	12.263**	1.160**
	$\pi_t - \tilde{\pi}_t$	0.004	1.829**	29.531**	3.460**	-5.614**	-0.284**	0.486	3.445**	9.919**	1.811**	24.345**	1.162**
	$y_t - \tilde{y}_t$	0.003	2.078**	197.632**	3.697**	135.635**	2.268**	0.425	3.091**	8.423**	1.637**	0.220**	0.023**
	volatility	0.004	2.078*	47.911**	3.077**	27.558**	1.225**	0.499	3.401**	101.265**	1.002**	158.734**	1.001**
Australia	$i_{t-1} - \tilde{i}_{t-1}$	0.002	1.677**	83.556**	2.140**	31.829**	0.605**	0.464	1.523*	10.528**	3.317**	6.704**	1.239**
	$y_t - \tilde{y}_t$	0.001	2.216**	77.310**	1.888**	38.603**	0.704**	0.261	1.942**	3.410**	1.568**	1.743**	0.343**
	$w_t - \tilde{w}_t(h)$	0.002	1.479*	74.386**	1.999**	19.521**	0.400**	0.391	1.486*	59.285**	3.136**	20.717**	0.686**

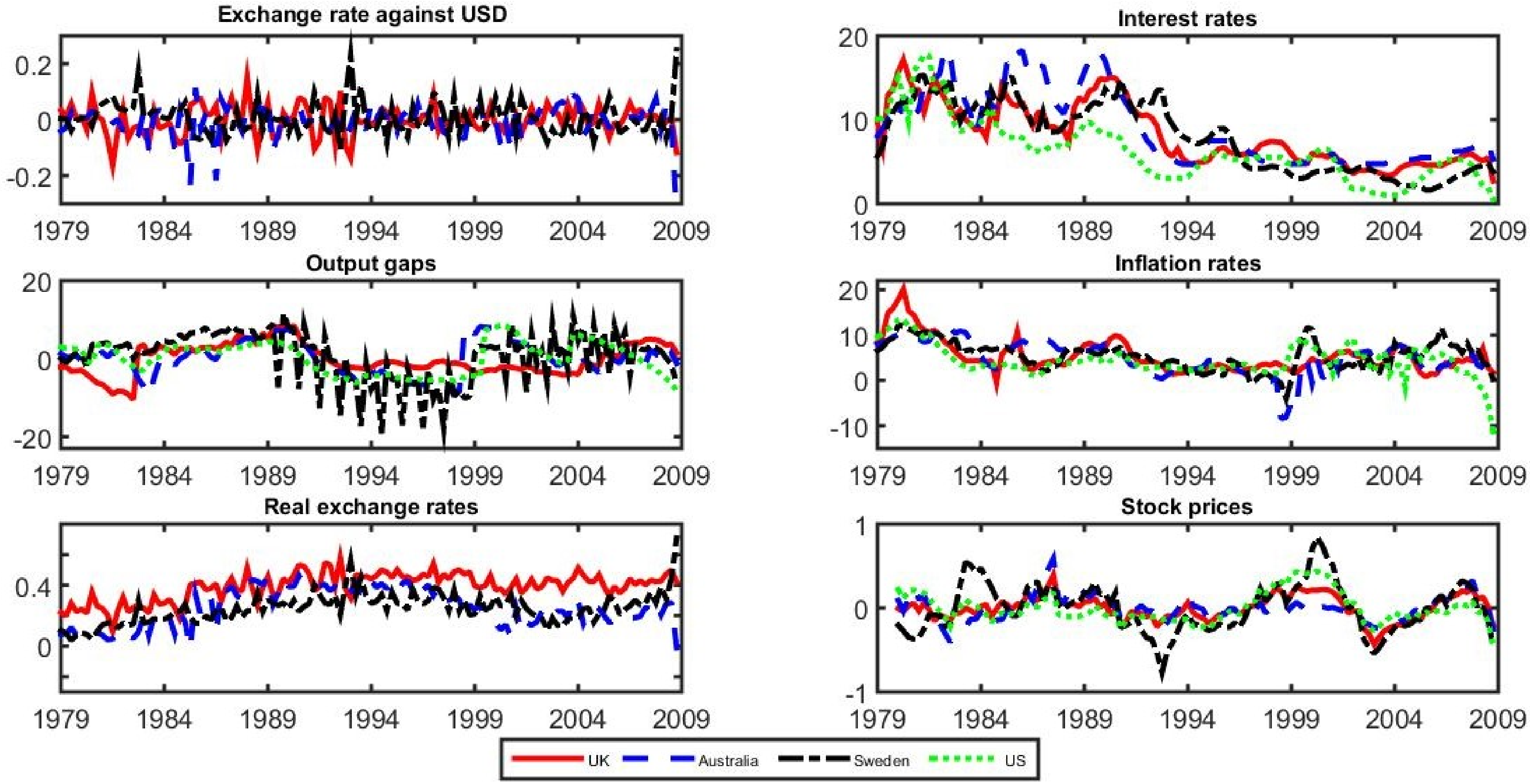
Notes: Panel A presents the comparison between the nonlinear model and the random walk, whereas Panel B presents the comparison between the nonlinear model with wealth effect and the linear model with wealth effect. Significance levels at 90% and 95% are denoted by one and two stars respectively. The Theil's U are the ratios of the MSPE between the nonlinear Taylor rule and corresponding benchmark, a value less than 1 means the nonlinear Taylor rule has a smaller MSPE. For CW statistics, the null hypothesis is rejected if the statistic is greater than $+1.282$ (for a one side 0.10 test) or $+1.645$ (for a one side 0.05 tests). The critical value for MSE-t, MSE-F, ENC-F and ENC-t are obtained from Clark and McCracken (2001) and McCracken (2004).

Table. 7. Forecasting results

		Panel A: Linear Model (Standard Taylor rule)						Panel B: Nonlinear UIP					
Country	***Transition variable***	***Theil's U***	***CW***	**ENC-F**	**ENC-t**	**MSE-F**	**MSE-t**	***Theil's U***	***CW***	**ENC-F**	**ENC-t**	**MSE-F**	**MSE-t**
UK	$y_t - \tilde{y}_t$	0.674	2.765**	13.003**	1.974**	5.175**	1.005**	**1.06**	2.966**	63.029**	4.747**	21.797**	1.200**
	volatility	0.548	2.503**	21.588**	2.899**	10.680**	0.880**	0.86	1.705**	62.118**	5.121**	9.505**	0.572**
	volatility15	0.566	2.951**	68.591**	4.451**	15.017**	0.677**	0.8	2.492**	78.956**	5.351**	20.963**	0.976**
Sweden	$i_{t-1} - \tilde{i}_{t-1}$	0.511	3.422**	10.079**	1.306*	9.650**	0.793**	0.95	1.648**	167.365**	3.494**	129.691**	2.384**
	$\pi_t - \tilde{\pi}_t$	0.519	3.253**	6.061**	3.028**	6.002**	1.893**	0.96	1.693**	37.887**	2.937**	5.646**	0.259**
	$y_t - \tilde{y}_t$	0.453	3.087**	19.300**	2.628**	8.920**	0.746**	0.84	2.609**	222.406**	3.295**	179.865**	2.407**
	volatility	0.532	3.273**	109.660**	1.002**	170.398**	1.001**	0.99	0.998	560.941**	1.020**	487.477**	0.880**
Australia	$i_{t-1} - \tilde{i}_{t-1}$	0.472	1.443*	39.667**	3.998**	2.178**	0.133**	0.73	1.893**	79.858**	1.665**	41.333**	0.657**
	$y_t - \tilde{y}_t$	0.266	1.775**	75.783**	2.058**	57.402**	1.191**	0.41	1.039	84.691**	1.673**	48.705**	0.735**
	$w_t - \tilde{w}_t(h)$	0.398	1.406*	8.916**	1.573**	14.950**	0.977**	0.62	1.535*	80.773**	1.776**	27.941**	0.480**

Notes: Panel A presents the comparison between the nonlinear model and the linear model, whereas Panel B presents the comparison between the nonlinear model and the non-linear UIP model. Significance levels at 90% and 95% are denoted by one and two stars respectively. The Theil's U are the ratios of the MSPE between the nonlinear Taylor rule and corresponding benchmark, a value less than 1 means the nonlinear Taylor rule has a smaller MSPE. For the CW statistics, the null hypothesis is rejected if the statistic is greater than $+1.282$ (for a one side 0.10 test) or $+1.645$ (for a one side 0.05 tests). The critical value for MSE-t, MSE-F, ENC-F and ENC-t are obtained from Clark and McCracken (2001) and McCracken (2004).

Figure 1. Plots of the data



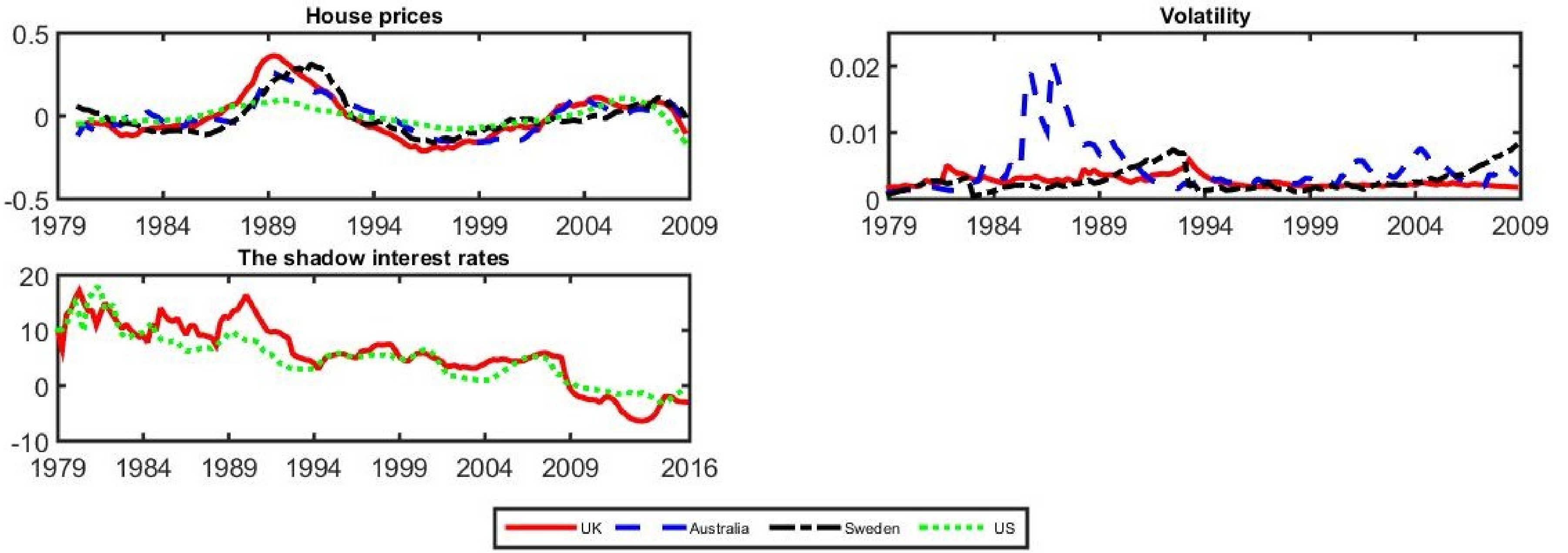

House prices
0.5
0
-0.5
1979
1984
1989
1994
1999
2004
2009
Volatility
0.02
0.01
0
1979
1984
1989
1994
1999
2004
2009
The shadow interest rates
20
10
0
-10
1979
1984
1989
1994
1999
2004
2009
2016
UK
Australia
Sweden
US

Figure 2. Estimated transition function over time

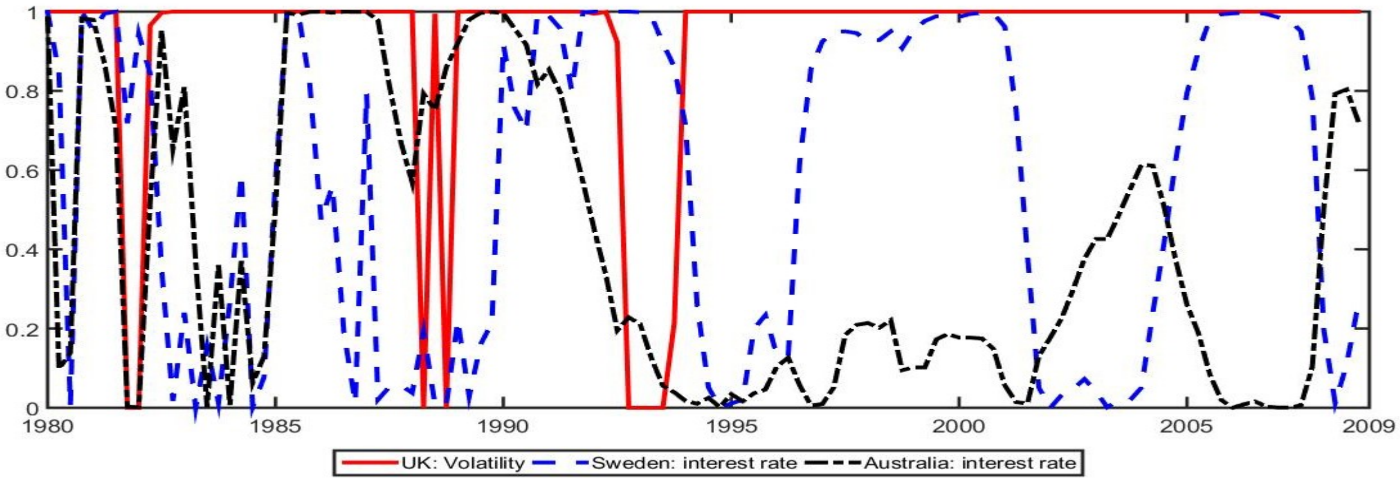


Figure 3. Estimated Transition Function

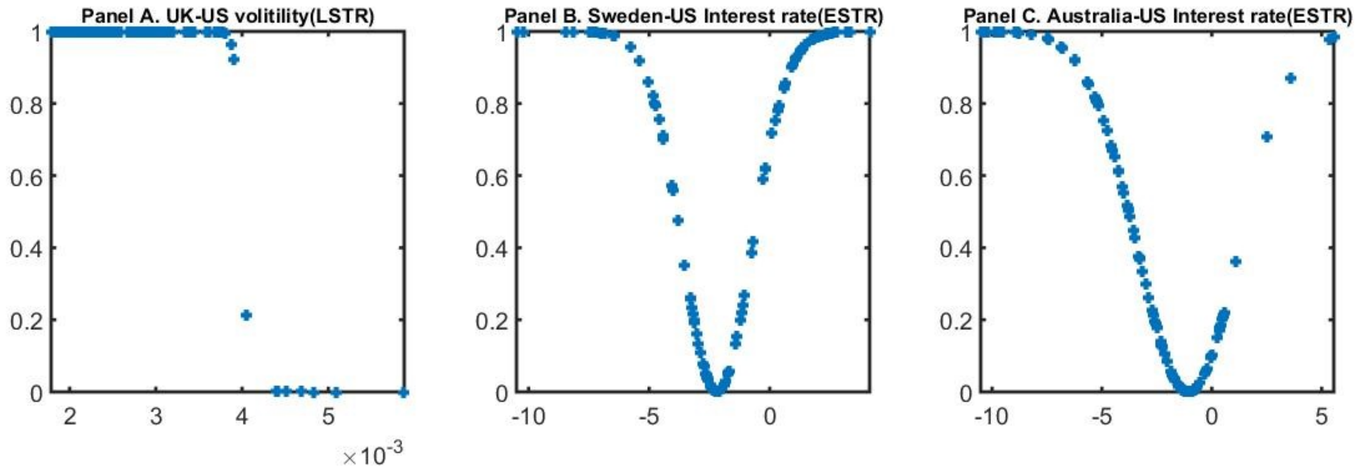
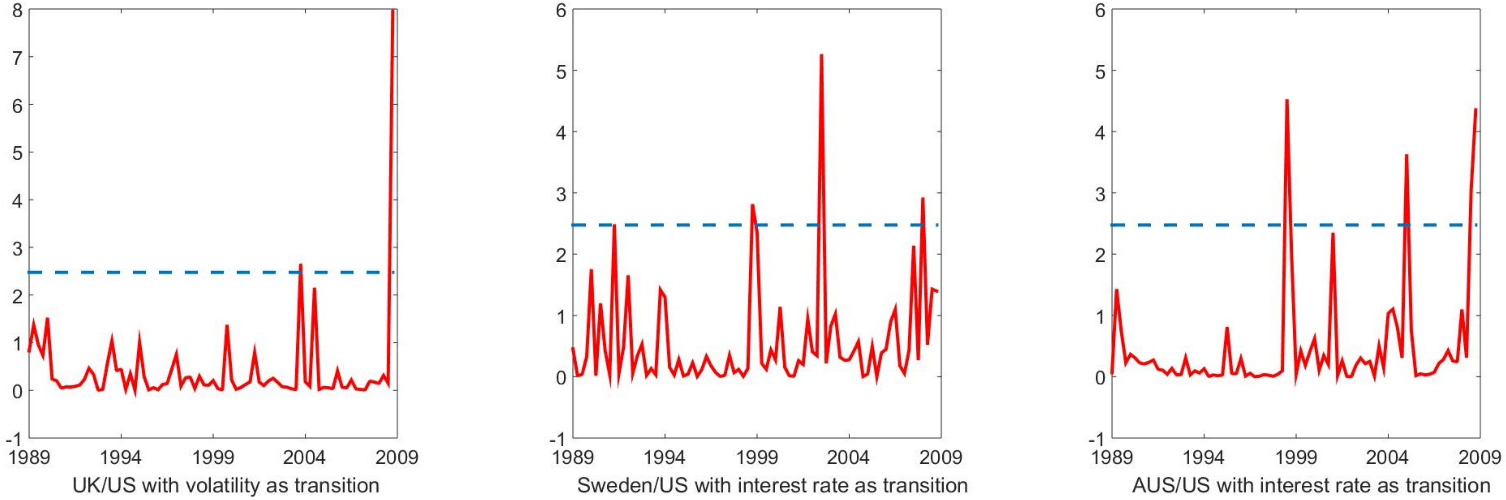


Figure 4. Fluctuation tests for the forecasts



Notes: this figure reports Giacomini and Rossi's (2010) Fluctuation test statistics (in absolute value) implemented using the Clark and West's (2006) statistics for comparing forecasts of the Non-linear Taylor rule exchange rate model relative to the Linear Taylor rule exchange rate models as the benchmark (solid line). The dashed line denotes the one-sided 5% critical value of the Fluctuation test statistic. If the estimated test statistic is below this line, the Taylor rule exchange rate model with stock prices forecasts significantly better than its benchmark.

Appendix

Linearity tests on the UIP model using data 1980Q1 -2008Q4

Transition variable ***($i_{t-1} - \tilde{i}_{t-1}$)***	H_0	H_{01}	H_{02}	H_{03}	***Type of model***
UK08	**0.0910***	**0.0045****	0.0467	0.6951	LSTR
UK15	**0.0985***	0.9332	0.2032	**0.0128****	LSTR
Sweden	**0.0014****	**0.0010****	0.0017	0.0627	LSTR
Australia	**0.0942***	0.9180	**0.0023****	0.0422	ESTR

Note: See Table